Logarithm Rules

The *base* b *logarithm* of a number is the *exponent* that we need to raise the *base* in order to get the number.

* [Logarithm definition](http://www.rapidtables.com/math/algebra/Logarithm.htm#log definition)
* [Logarithm rules](http://www.rapidtables.com/math/algebra/Logarithm.htm#log rules)
* [Logarithm problems](http://www.rapidtables.com/math/algebra/Logarithm.htm#problems)
* [Graph of log(x)](http://www.rapidtables.com/math/algebra/Logarithm.htm#log-graph)
* [Logarithm table](http://www.rapidtables.com/math/algebra/Logarithm.htm#log table)
* [Logarithm calculator](http://www.rapidtables.com/calc/math/Log_Calculator.htm)

Logarithm definition

When b is raised to the power of y is equal x:

*by* = *x*

Then the base b logarithm of x is equal to y:

log*b*(*x*)*= y*

For example when:

24 = 16

Then

log2(16) = 4

Logarithm as inverse function of exponential function

The logarithmic function,

*y*= log*b*(*x*)

is the inverse function of the exponential function,

*x*=*by*

So if we calculate the exponential function of the logarithm of x (x>0),

*f*(*f*-1(*x*)) = *b*log*b*(*x*) = *x*

Or if we calculate the logarithm of the exponential function of x,

*f*-1(*f*(*x*)) = log*b*(*bx*) = *x*

Natural logarithm (ln)

[Natural logarithm](http://www.rapidtables.com/math/algebra/Ln.htm) is a logarithm to the base e:

ln(*x*) = log*e*(*x*)

When [e constant](http://www.rapidtables.com/math/number/e_constant.htm) is the number:

e=\lim_{x\to \infty }(1+\frac{1}{n})^{n}=\lim_{x\to 0 }(1+n)^\frac{1}{n}=2.71828183...

See: [Natural logarithm](http://www.rapidtables.com/math/algebra/Ln.htm)

Inverse logarithm calculation

The inverse logarithm (or anti logarithm) is calculated by raising the base b to the logarithm y:

*x* = log-1(*y*) = *by*

Logarithmic function

The logarithmic function has the basic form of:

*f*(*x*) = log*b*(*x*)

Logarithm rules

|  |  |
| --- | --- |
| Rule name | Rule |
| [**Logarithm product rule**](http://www.rapidtables.com/math/algebra/Logarithm.htm#product rule) | log*b*(*x ∙ y*) = log*b*(*x*)*+*log*b*(*y*) |
| [**Logarithm quotient rule**](http://www.rapidtables.com/math/algebra/Logarithm.htm#quotient rule) | log*b*(*x / y*) = log*b*(*x*)*-*log*b*(*y*) |
| [**Logarithm power rule**](http://www.rapidtables.com/math/algebra/Logarithm.htm#power rule) | log*b*(*x y*) = *y ∙*log*b*(*x*) |
| [**Logarithm base switch rule**](http://www.rapidtables.com/math/algebra/Logarithm.htm#base switch) | log*b*(*c*) = 1 / log*c*(*b*) |
| [**Logarithm base change rule**](http://www.rapidtables.com/math/algebra/logarithm/Logarithm_Base_Change.htm) | log*b*(*x*) = log*c*(*x*) / log*c*(*b*) |
| [**Derivative of logarithm**](http://www.rapidtables.com/math/algebra/Logarithm.htm#log derivative) | *f*(*x*) = log*b*(*x*)⇒ *f '*(*x*) = 1 / (*x* ln(*b*) ) |
| [**Integral of logarithm**](http://www.rapidtables.com/math/algebra/Logarithm.htm#log integral) | ∫log*b*(*x*) *dx* = *x ∙*( log*b*(*x*)- 1 / ln(*b*)) + *C* |
| [**Logarithm of negative number**](http://www.rapidtables.com/math/algebra/Logarithm.htm#log negative) | log*b*(*x*)is undefined when *x*≤ 0 |
| [**Logarithm of 0**](http://www.rapidtables.com/math/algebra/Logarithm.htm#log 0) | log*b*(0) is undefined |
| \lim_{x\to 0^+}\textup{log}_b(x)=-\infty |
| [**Logarithm of 1**](http://www.rapidtables.com/math/algebra/Logarithm.htm#log 1) | log*b*(1) = 0 |
| [**Logarithm of the base**](http://www.rapidtables.com/math/algebra/Logarithm.htm#log b) | log*b*(*b*) = 1 |
| [**Logarithm of infinity**](http://www.rapidtables.com/math/algebra/Logarithm.htm#log b) | lim log*b*(*∞*) = *∞,*when*x*→∞ |

See: [Logarithm rules](http://www.rapidtables.com/math/algebra/logarithm/Logarithm_Rules.htm)

**Logarithm product rule**

The logarithm of the multiplication of x and y is the sum of logarithm of x and logarithm of y.

log*b*(*x ∙ y*) = log*b*(*x*)*+*log*b*(*y*)

For example:

log10(3*∙*7) = log10(3)*+*log10(7)

**Logarithm quotient rule**

The logarithm of the division of x and y is the difference of logarithm of x and logarithm of y.

log*b*(*x / y*) = log*b*(*x*)*-*log*b*(*y*)

For example:

log10(3*/*7) = log10(3)*-*log10(7)

**Logarithm power rule**

The logarithm of x raised to the power of y is y times the logarithm of x.

log*b*(*x y*) = *y ∙*log*b*(*x*)

For example:

log10(28) = 8*∙*log10(2)

**Logarithm base switch rule**

The base b logarithm of c is 1 divided by the base c logarithm of b.

log*b*(*c*) = 1 / log*c*(*b*)

For example:

log2(8) = 1 / log8(2)

**Logarithm base change rule**

The base b logarithm of x is base c logarithm of x divided by the base c logarithm of b.

log*b*(*x*) = log*c*(*x*) / log*c*(*b*)

For example, in order to calculate log2(8) in calculator, we need to change the base to 10:

log2(8) = log10(8) / log10(2)

See: [log base change rule](http://www.rapidtables.com/math/algebra/logarithm/Logarithm_Base_Change.htm)

**Logarithm of negative number**

The base b real logarithm of x when x<=0 is undefined when x is negative or equal to zero:

log*b*(*x*) is undefined when *x* ≤ 0

See: [log of negative number](http://www.rapidtables.com/math/algebra/logarithm/Logarithm_of_Negative_Number.htm)

**Logarithm of 0**

The base b logarithm of zero is undefined:

log*b*(0) is undefined

The limit of the base b logarithm of x, when x approaches zero, is minus infinity:

\lim_{x\to 0^+}\textup{log}_b(x)=-\infty 

See: [log of zero](http://www.rapidtables.com/math/algebra/logarithm/Logarithm_of_0.htm)

**Logarithm of 1**

The base b logarithm of one is zero:

log*b*(1) = 0

For example, teh base two logarithm of one is zero:

log2(1) = 0

See: [log of one](http://www.rapidtables.com/math/algebra/logarithm/Logarithm_of_1.htm)

**Logarithm of infinity**

The limit of the base b logarithm of x, when x approaches infinity, is equal to infinity:

lim log*b*(*x*) = ∞, when *x*→∞

See: [log of infinity](http://www.rapidtables.com/math/algebra/logarithm/Logarithm_of_Infinity.htm)

**Logarithm of the base**

The base b logarithm of b is one:

log*b*(*b*) = 1

For example, the base two logarithm of two is one:

log2(2) = 1

**Logarithm derivative**

When

*f*(*x*) = log*b*(*x*)

Then the derivative of f(x):

*f '*(*x*) = 1 / (*x* ln(*b*) )

See: [log derivative](http://www.rapidtables.com/math/algebra/logarithm/Logarithm_Derivative.htm)

**Logarithm integral**

The integral of logarithm of x:

∫log*b*(*x*) *dx* = *x ∙*( log*b*(*x*)- 1 / ln(*b*)) + *C*

For example:

∫log2(*x*) *dx* = *x ∙*( log2(*x*)- 1 / ln(2)) + *C*

Logarithm approximation

log2(*x*) ≈ *n* + (*x*/2*n* - 1) ,

Logarithm problems and answers

**Problem #1**

Find x for

log2(*x*) + log2(*x*-3) = 2

**Solution:**

Using the product rule:

log2(*x∙*(*x*-3)) = 2

Changing the logarithm form according to the logarithm definition:

*x∙*(*x*-3) = 22

Or

*x*2-3*x*-4 = 0

Solving the quadratic equation:

*x*1,2 = [3±√(9+16) ] / 2 = [3±5] / 2 = 4,-1

Since the logarithm is not defined for negative numbers, the answer is:

*x* = 4

**Problem #2**

Find x for

log3(*x*+2) - log3(*x*) = 2

**Solution:**

Using the quotient rule:

log3((*x*+2) /*x*) = 2

Changing the logarithm form according to the logarithm definition:

(*x*+2)/*x* = 32

Or

*x*+2 = 9*x*

Or

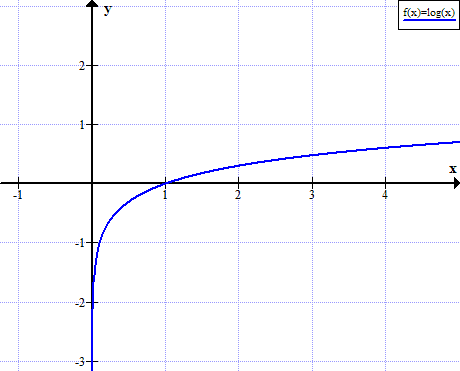
8*x* = 2

Or

*x* = 0.25

Graph of log(x)

log(x) is not defined for real non positive values of x:



Logarithms table

|  |  |  |  |
| --- | --- | --- | --- |
| *x* | log10*x* | log2*x* | log*ex* |
| 0 | undefined | undefined | undefined |
| 0+ | - ∞ | - ∞ | - ∞ |
| 0.0001 | -4.000000 | -13.287712 | -9.210340 |
| 0.001 | -3.000000 | -9.965784 | -6.907755 |
| 0.01 | -2.000000 | -6.643856 | -4.605170 |
| 0.1 | -1.000000 | -3.321928 | -2.302585 |
| 1 | 0.000000 | 0.000000 | 0.000000 |
| 2 | 0.301030 | 1.000000 | 0.693147 |
| 3 | 0.477121 | 1.584963 | 1.098612 |
| 4 | 0.602060 | 2.000000 | 1.386294 |
| 5 | 0.698970 | 2.321928 | 1.609438 |
| 6 | 0.778151 | 2.584963 | 1.791759 |
| 7 | 0.845098 | 2.807355 | 1.945910 |
| 8 | 0.903090 | 3.000000 | 2.079442 |
| 9 | 0.954243 | 3.169925 | 2.197225 |
| 10 | 1.000000 | 3.321928 | 2.302585 |
| 20 | 1.301030 | 4.321928 | 2.995732 |
| 30 | 1.477121 | 4.906891 | 3.401197 |
| 40 | 1.602060 | 5.321928 | 3.688879 |
| 50 | 1.698970 | 5.643856 | 3.912023 |
| 60 | 1.778151 | 5.906991 | 4.094345 |
| 70 | 1.845098 | 6.129283 | 4.248495 |
| 80 | 1.903090 | 6.321928 | 4.382027 |
| 90 | 1.954243 | 6.491853 | 4.499810 |
| 100 | 2.000000 | 6.643856 | 4.605170 |
| 200 | 2.301030 | 7.643856 | 5.298317 |
| 300 | 2.477121 | 8.228819 | 5.703782 |
| 400 | 2.602060 | 8.643856 | 5.991465 |
| 500 | 2.698970 | 8.965784 | 6.214608 |
| 600 | 2.778151 | 9.228819 | 6.396930 |
| 700 | 2.845098 | 9.451211 | 6.551080 |
| 800 | 2.903090 | 9.643856 | 6.684612 |
| 900 | 2.954243 | 9.813781 | 6.802395 |
| 1000 | 3.000000 | 9.965784 | 6.907755 |
| 10000 | 4.000000 | 13.287712 | 9.210340 |

When

*ey* = *x*

Then base e logarithm of x is

ln(*x*) = log*e*(*x*)*= y*

loga x = N means that aN= x